The physical observer I: Absolute and relative fields

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Abstract

Quantum Jet Theory (QJT) is a deformation of QFT where also the quantum dynamics of the observer is taken into account. This is achieved by introducing relative fields, labelled by locations measured by rods relative to the observer's position. In the Hamiltonian formalism, the observer's momentum is modified: $p_i \to p_i - P_i$, where P_i is the momentum carried by the field quanta. The free scalar field, free electromagnetism and gravity are treated as examples. Standard QFT results are recovered in the limit that the observer's mass $M \to \infty$ and its charge $e \to 0$. This limit is well defined except for gravity, because $e = M$ in that case (heavy mass equals inert mass). In a companion paper we describe how QJT also leads to new observer-dependent gauge and diff anomalies, which can not be formulated within QFT proper.

1 Physical motivation

Every experiment is an interaction between a system and a detector, and the result depends on the physical properties of both. We want to eliminate the detector dependence as far as possible, but not further than that. The main thesis in this paper is that the detector's mass can not be eliminated in the presence of gravity.

Every physical detector[1] has some charge e (because otherwise it can not interact with the system and thus not observe it) and some finite and nonzero mass M. To extract the detector-independent physics, we want to take the joint limit $e \to 0$ (so the detector does not perturb the fields) and $M \to \infty$ (so the detector follows a well-defined, classical path in spacetime). If $M \neq \infty$, the detector's position and velocity at the same instant do not commute, and hence its worldline is subject to quantum fluctuations.

The joint limit $e \to 0$, $M \to \infty$ is described by QFT. This limit is well defined for non-gravitational interactions, but runs into serious trouble when gravity is taken into account. The reason is that the gravitational charge is closely related to mass – the heavy mass equals the inert mass. This observation immediately leads to a successful postdiction: QFT is incompatible with gravity.

To remedy this problem, we propose to replace QFT by a more complete theory, which explicitly describes both the quantum-mechanical detector and the quantized fields. This theory is, or will be, called Quantum Jet Theory (QJT). The strategy for doing this is to introduce the detectors's quantized worldline, defined operationally by the readings of clocks and rods. All fields are expanded a Taylor series around the detector's worldline, and physics is formulated in terms of the Taylor coefficients rather than in terms of the fields themselves. The motivates the name QJT, because in mathematics a jet is essentially the same thing as a Taylor expansion[2].

Historically, QJT grew out of the projective representation theory of the algebra of spacetime diffeomorphisms, i.e. the multi-dimensional Virasoro algebra. In particular, it was noted in [1] that off-shell representations of lowest-energy type must be formulated in terms of trajectories of jets rather than in terms of the spacetime fields themselves. Several flawed attempts to apply this insight to physics were made, cf. [2, 3]. In [4], the correct treatment was finally found, at least for the harmonic oscillator; the term QJT was also coined in that paper.

Unfortunately, the formalism developed in those papers was quite unwieldy, for two reasons. First, working with jets rather than fields is complicated by the fact that jets do not mix nicely with nonlocal integrals such as the action functional or the Hamiltonian. Second, I attempted to do canon-

[1] We will use the words "detector" and "observer" synonymously. This does of course not imply that the observer is human or even animate.

[2] More precisely, a p-jet is an equivalence class of functions; two functions belong to the same class if all derivatives up to order p, evaluated at some point q, are the same. Since a p-jet has a unique representative which is a polynomial of order at most p, namely the truncated Taylor series around q, we may canonically identify jets and Taylor expansions.

ical quantization in a manifestly covariant way, identifying phase space with the space of histories. It is the purpose of the present paper to simplify the formulation of QJT by avoiding these complications. We break manifest covariance, and work with the fields themselves regarded as generating functions for their Taylor series. This makes it far simpler to extract the physical content of QJT.

In a companion paper [5], we discuss the appearance of gauge and diff anomalies. Contrary to QFT, gauge anomalies are a generic feature in QJT. The relevant cocycles expicitly depend on the observer's trajectory, and can hence not be formulated within QFT, where the observer is never introduced. Because these new gauge anomalies have an unconventional form (e.g., for Yang-Mills theory they are proportional to the second Casimir rather than to the third), standand intuition about gauge anomalies does not apply; in particular, these gauge anomalies do not necessarily render the theory inconsistent.

2 Partial and complete observables in QFT and QJT

Recall the distinction between partial and complete variables made by Rovelli [8]:

- *Partial observable*: a physical quantity to which we can associate a (measuring) procedure leading to a number.
- *Complete observable*: a quantity whose value can be predicted by the theory (in classical theory); or whose probability distribution can be predicted by the theory (in quantum theory).

A physical experiment always consists of at least two measurements: the reading of the detector and the reading of a clock. The partial observables are thus $\{A, t\}$, where A is the quantity of interest and t is time. Although both partial observables can be measured, neither can be predicted without knowledge about the other. What can be predicted is the complete observable $A(t)$, the value of A at time t. Of course, we can only make predictions, or check that our assumed dynamics is correct, provided that we know the state of the system. Hence we must first measure the partial observables $\{A, t\}$ sufficiently many times to determine the state. Once that is done, the outcome of further observations is predicted by the theory.

Complete observables correspond to self-adjoint operators in quantum mechanics, partial observables do not. The complete observable $A(t)$ is

the value of A at time t; since the measurement of this value is subject to quantum flucuations, it is described by an operator. In contrast, time t itself is a partial observable which serves to localize the experiment in time, and as such it is a c-number parameter. Perhaps the distinction is made clearest by the questions answered by the different types of observables: $A(t)$ answers "What is the value of A at time t?" and t (or $t(t)$) answers "What is the value of time at time t?". Clearly, the answer to the second question has no room for quantum fluctuations, and hence it is given by a c-number parameter rather than an operator. In my opinion, this settles the apparent paradox with Pauli's theorem [6], which asserts that there can be no time operator in quantum mechanics (provided that the energy is bounded from below).

More symmetrically, a complete observable is a correlation (A, t) between partial observables. If the relation between A and t is monotonous, we can regard this complete observable either as the value $A(t)$ of A at time t, or the value $t(A)$ of t at detector reading A. Either way the complete observable is subject to quantum fluctuations and thus given by an operator. The monotony assumption means that A is another clock. E.g., t could be the observer's proper time τ (the ticks of the local clock), whereas A could be reference time broadcasted from a GPS (Global Positioning System) satellite. If the experiment described by A amounts to the detection of one of these broadcasted signals, it may be viewed as another definition of time, and presumably a more accurate one than the reading of a local clock device. The observed value of broadcasted time at a given observed value of the local clock is clearly subject to quantum fluctuations, and hence described by an operator.

Let us now turn to field theory. In QFT the complete observables are fields $\phi(x)$, where $x = x^\mu$ is a spacetime point. In any experiment, we hence measure two kinds of partial observables:

ϕ: The value of the field, measured by our experimental apparatus.

x^μ: The detector's spacetime location, measured by rods and clocks.

Rather than using rods and clocks, x^μ can more conveniently be measured using GPS receivers; we will therefore refer to x^0 as *GPS time* and x^i as *GPS position* [7].

However, there are subtle physical problems with using the complete observables (ϕ, x). The first problem is that we need to know the state of the system in order to make predictions, and infinitely many observations are required to determine the state uniquely. Typically, we must determine the

values of the field throughout an equal-time surface, say $x^0 = 0$. Rovelli suggests that one should avoid this problem by making additional assumptions about the state [8], something which I find unattractive.

A second problem is that a single detector can only measure the field at a single point on a simultaneity surface, namely where its worldline intersects the surface. Hence we need an array of detectors, each equipped with a separate GPS receiver. However, at time $t = 0$ the master detector can not know about the full state at this time; only at some later time $t = T$, when the information from the most distant slave detector has reached the master, can the full state back at $t = 0$ be known. Moreover, to determine the identity of an individual detector, we need to measure a new partial observable n, which can not be specified to arbitrary precision if the spacetime location x is a c-number. Starting from the partial observables $\{n, x, \phi\}$, we can may take x as the independent variable, hence a c-number. Then the complete observable $n(x)$, which tells us which one of the detectors is located at x, is an operator. Alternatively, we can ask about the measurement in detector n, but then the complete observable $x(n)$, the location of this particular detector, becomes an operator. Thus the spacetime location and the detector's identity can not be simultaneously specified to arbitrary precision.

Finally, since the detector's position q^i is a partial observable, it can only be measured but not predicted. This is obviously incorrect for physical detectors, which move according to some equations of motion. From a physical point of view, the difference between QJT and QFT is that the former takes the detector's nontrivial quantum dynamics into account[3].

In QJT we have three types of partial observables:

- ϕ: The value of the field, measured by our experimental apparatus.
- q^μ: The detector's spacetime position, measured by its GPS receiver. As the notation indicates, q^μ is assumed to transform as a spacetime vector.
- τ: The detector's proper time, measured by a local clock.

[3] This difference can be illustrated by the following exaggerated example. Consider an experiment at the LHC. We can measure the detector's location (close to Geneva), its velocity (close to zero), and time (year 2008). These are partial observables which serve to localize the fields, and within QFT nothing can be said about the detector's location in the future. In contrast, QJT also describes the detector's dynamics, and hence the detector's position $q^i(t)$ is a complete observable whose future values can be predicted (it is likely to remain close to Geneva).

Note that we have two time observables, proper time τ and GPS time q^0. From this set of partial observables, we can construct two types of complete observables: $\phi(\tau)$, the reading of the detector when proper time is τ, and $q^\mu(\tau)$, the reading of the GPS receiver when proper time is τ. Unlike the situation in QFT, these observables can be measured by a single detector, so we have no problems with nonlocality.

There is of course nothing special about proper time. We can (and eventually will) instead use GPS time q^0 as our independent time variable, which makes proper time $\tau(q^0)$ into a complete observable. To make the treatment more symmetrical, we introduce an arbitrary timelike parameter t as our independent variable. The QJT observables then become functions of t: $q^\mu(t)$, $\tau(t)$ and $\phi(t)$. The timelike parameter t is not physical since the theory now has a gauge symmetry of reparametrizations of the observer's trajectory; the reparametrization generators $L(t)$ obey the Witt algebra

$$[L(t), L(t')] \quad = \quad (L(t) + L(t'))\dot{\delta}(t - t') \tag{2.1}$$

The QJT observables considered so far evidently contain much less information than QFT observables, because they only know about the field along the detector's worldline. In particular, we can not predict anything, because every field theory prediction involves partial derivatives transverse to the observer's trajectory[4]. Fortunately this problem can easily be solved by measuring further local data. The most general complete observable that a local detector can measure at time t is not just the field $\phi(t)$ itself, but also the gradient $\phi_{,\mu}(t) = \partial_\mu\phi(x)|_{x=q(t)}$, as well as higher partial derivatives of the field such as $\phi_{,\mu\nu}(t)$. Clearly, the new observables constructed in this way are not all independent. There are constraints relating derivatives in the temporal direction to time evolution, the simplest one being

$$\dot{\phi}(t) = \dot{q}^\mu(t)\phi_{,\mu}(t). \tag{2.2}$$

We can use this equation to eliminate one component of the gradient. If we use the reparametrization symmetry (2.1) to fix $q^0(t) = t$, all partial derivatives with at least one $\mu = 0$ index can be eliminated by the relations

$$\phi_{,0\mu_1..\mu_p}(t) = \dot{\phi}_{,\mu_1..\mu_p}(t) - \dot{q}^i(t)\phi_{,i\mu_1..\mu_p}(t), \tag{2.3}$$

which are analogous to (2.2). The QJT observables $\phi_{,\mu_1..\mu_p}(t)$ contain the same amount information as the QFT observables $\phi(x)$ in a neighborhood of the observer's trajectory $q^\mu(t)$.

[4]The obvious exception is when the number of spatial dimensions $d = 0$, i.e. quantum mechanics. Like QFT, QJT reduces to quantum mechanics in this case, because the observer's location can not fluctuate when space consists of a single point.

Let us summarize the main differences between QFT and QJT:

- In QFT, the partial observables are $\{x^\mu, \phi\}$, and the complete observables $\phi(x)$ answer the question: "What does the detector measure when the GPS receiver measures x^μ?". The answer requires an array of detectors, whose identity and position can not both be measured sharply, and whose time evolution can not be predicted.
- In QJT, the partial observables are $\{t, \tau, q^\mu, \phi, \phi_{,\mu}, \phi_{,\mu\nu}, ...\}$. The complete observables $\tau(t)$, $q^\mu(t)$, $\phi(t)$, $\phi_{,\mu}(t)$, ... answer the questions: "What do the local clock, the GPS receiver and the field detector measure when the time parameter is t?". This can be answered by a single, local detector, whose position evolves in time in a predictable manner.

3 Absolute and relative fields

3.1 Spacetime fields

We now start with the formalization of the physical discussion in the previous section. Consider some field $\phi(x)$ over $(d+1)$-dimensional spacetime, where $x = (x^\mu) \in \mathbb{R}^{d+1}$ are spacetime coordinates. In QFT, these coordinates are measured relative some absolute, fixed origin[5]. To emphasize this point, we call this an *absolute field* $\phi_A(x)$. In QJT we instead consider the *relative field* $\phi_R(x, t)$, where the spacetime coordinates are measured relative to the physical observer's spacetime location $q^\mu(t)$. The time coordinate t will soon be identified with GPS time: $t = x^0 = q^0(t)$. The important difference is that $q^i(t)$ is "on the other side of the rod", i.e. that positions are measured relative to the observer's location rather than relative to some fixed origin. The observer's position at time t can be predicted once we know the observer's quantum dynamics, and hence $q^i(t)$ must be a complete observable, which becomes an operator after quantization. This is the essential novelty in QJT.

Absolute and relative fields are related by

$$\begin{aligned} \phi_R(x,t) &= \phi_A(x + q(t)), \\ \phi_A(x) &= \phi_R(x - q(t), t), \end{aligned} \tag{3.1}$$

[5]Physically, the fixed origin may be thought of as the location of an infinitely heavy observer, e.g. a GPS satellite.

because absolute and relative position are related by $x_A = x_R + q(t)$. As the notation indicates, absolute fields do not depend on the time parameter t. From $d\phi_A(x)/dt = 0$ it follows that relative fields satisfy

$$\frac{\partial}{\partial t}\phi_R(x,t) - \dot{q}^\mu(t)\partial_\mu\phi_R(x,t) \equiv 0. \tag{3.2}$$

In QFT we are used to operator-valued functions like $\phi_A(x)$, which depends on c-number arguments, but what is the meaning of $\phi_R(x,t) = \phi_A(x + q(t))$, where the argument is itself an operator? To answer this question, we consider absolute fields being defined by their Taylor series, *viz.*

$$\phi_A(x) = \sum_{m\in\mathbb{N}^{d+1}} \frac{1}{m!}\phi_{,m}(t)(x - q(t))^m. \tag{3.3}$$

Here we use standard multi-index notation introduced e.g. in [2]: $m = (m_0, m_1, ..., m_d) \in \mathbb{N}^{d+1}$, $m! = m_0!m_1!...m_d!$, $(x - q)^m = (x^0 - q^0)^{m_0}(x^1 - q^1)^{m_1}...(x^d - q^d)^{m_d}$. Denote by $\hat{\mu}$ a unit vector in the μ:th direction, so that $m + \hat{\mu} = (m_0, ..., m_\mu + 1, ..., m_d)$. The Taylor cooefficients

$$\phi_{,m}(t) = \partial_m\phi_A(q(t),t) = \underbrace{\partial_0..\partial_0}_{m_0}\underbrace{\partial_1..\partial_1}_{m_1}..\underbrace{\partial_d..\partial_d}_{m_d}\phi_A(q(t),t) \tag{3.4}$$

can be identified with the $|m|$:th order derivative of $\phi_A(x,t)$, evaluated on the observer's trajectory $q^\mu(t)$. Note the difference between $m = 0$ and $m = \hat{0}$: $\phi_{,0}(t) = \phi_A(q(t))$ but $\phi_{,\hat{0}}(t) = \partial_0\phi_A(q(t))$.

Analogously, the corresponding relative field is defined as the MacLaurin series

$$\phi_R(x) = \sum_{m\in\mathbb{N}^{d+1}} \frac{1}{m!}\phi_{,m}(t)x^m. \tag{3.5}$$

The identity (3.2) becomes

$$\dot{\phi}_{,m}(t) - \sum_{\mu=0}^{d} \dot{q}^\mu(t)\phi_{,m+\hat{\mu}}(t) = 0, \tag{3.6}$$

which contains (2.2) as a special case. Although we will not work explicitly with the series (3.3) and (3.5) in this paper, they are useful as an unambiguous definition of fields with operator-valued arguments. All formulas for the fields can readily be transformed into hierarchies of equations for the Taylor coefficients in (3.3) or (3.5). This is the motivation for the name QJT (Quantum Jet Theory).

The absolute field $\phi_A(x)$ depends on two types of coordinates:

- x^0 GPS time relative to a fixed origin. This is a partial observable and hence a c-number parameter.
- x^i GPS position relative to a fixed origin. This is a partial observable and hence a c-number parameter.

These coordinates can be combined into a spacetime vector $x^\mu = (x^0, x^i)$.

In contrast, we can consider no less than six different spacetime coordinates for the relative field $\phi_R(x,t)$:

- t An arbitrary timelike gauge parameter, which can be eliminated by gauge-fixing the reparametrization symmetry (2.1).
- $q^0(t)$ The observer's GPS time relative to a fixed origin, at parameter time t. This is a complete observable once the observer's dynamics is specified, and hence an operator.
- $q^i(t)$ The observer's GPS position relative to a fixed origin, at parameter time t. This is a complete observable once the observer's dynamics is specified, and hence an operator.
- x^0 GPS time relative to the observer's time $q^0(t)$. This is a partial observable and hence a c-number parameter.
- x^i GPS position relative to the observer's position $q^i(t)$. This is a partial observable and hence a c-number parameter.
- $\tau(t)$ The observer's proper time, as measured by a local clock, at parameter time t. It depends on the metric $g_{\mu\nu}$ as $\tau(t)^2 = g^A_{\mu\nu}(q(t))\dot q^\mu(t)\dot q^\nu(t)$ (absolute field) or $\tau(t)^2 = g^R_{\mu\nu}(0)\dot q^\mu(t)\dot q^\nu(t) = g_{\mu\nu,0}(t)\dot q^\mu(t)\dot q^\nu(t)$ (relative field).

These coordinates are combined into two spacetime vectors $q^\mu(t) = (q^0(t), q^i(t))$ and $x^\mu = (x^0, x^i)$, whereas proper time τ is a Lorentz scalar.

3.2 Space-time decomposition

By definition (3.1), the relative field $\phi_R(x,t)$ depends on three time variables x^0, t and $q^0(t)$. This is clearly two too many, and this will lead to various complications, e.g. the well-known type of gauge symmetry associated with parametrized time. Since we do not wish to deal with this type of complication here, but rather want to extract the physical consequences of relative fields, we eliminate the two extra time variables.

First use the reparametrization freedom to equal the time parameter to the detector's GPS time,

$$q^0(t) = t. \tag{3.7}$$

Moreover, we foliate spacetime into slices of constant GPS time,

$$x^0 = t. \tag{3.8}$$

The absolute field (3.3) then takes the form

$$\phi_A(t, \mathbf{x}) = \sum_{\mathbf{m}\in\mathbb{N}^d} \frac{1}{\mathbf{m}!} \phi_{,\mathbf{m}}(t)(\mathbf{x} - \mathbf{q}(t))^{\mathbf{m}}, \tag{3.9}$$

where boldface denotes d-dimensional spatial vectors, e.g. $\mathbf{x} = (x^i)$, $\mathbf{m} = (m_i)$. As usual, greek indices μ, ν run over spacetime directions and latin indices i, j label space directions. In (3.9) there is a single time coordinate t. The pair $(t, \mathbf{x})$ is enough to uniquely label the spacetime point where the field ϕ_A is measured, and hence they are partial, c-number observables. The relative time coordinate becomes $x^0 = 0$, and the relative field (3.5) becomes

$$\phi_R(t, \mathbf{x}) = \sum_{\mathbf{m}\in\mathbb{N}^d} \frac{1}{\mathbf{m}!} \phi_{,\mathbf{m}}(t)\mathbf{x}^{\mathbf{m}}. \tag{3.10}$$

Both conditions (3.7) and (3.8) break manifest Lorentz symmetry. The latter is just the ordinary foliation in the Hamiltonian formalism, and hence it does not break true Lorentz invariance. In contrast, the former condition has no counterpart in QFT, and it is possible that it also breaks true Lorentz symmetry. This is not unphysical, because in QJT there is a distinguished direction in spacetime, namely parallel to the physical observer's trajectory.

In general-covariant theories it is likely that if manifest diffeomorphism invariance is broken, so is true diffeomorphism invariance. The foliation (3.8) is problematic already in QFT, because the notion of an equal-time surface depends on the quantized metric. Moreover, the assumption (3.7) was studied in a diffeomorphism algebra context in [1], section 7. Although it does not affect the spacetime diffeomorphism algebra proper, reparametrization cocycles associated with the Witt algebra (2.1) transmute into complicated diffeomorphism cocycles, which are noncovariant because they single out the x^0 direction.

From (3.2) and $\dot q^0(t) = 1$ we find that

$$\begin{aligned} \partial_0\phi_A(t, \mathbf{x}) &= \dot\phi_A(t, \mathbf{x}), \\ \partial_0\phi_R(t, \mathbf{x}) &= \dot\phi_R(t, \mathbf{x}) - \dot q^i(t)\partial_i\phi_R(t, \mathbf{x}), \end{aligned} \tag{3.11}$$

where we denote the partial derivative with respect to t by a dot:

$$\dot{\phi}_R(t, \mathbf{x}) \equiv \frac{\partial}{\partial t} \phi_R(t, \mathbf{x}). \tag{3.12}$$

The expansions (3.9) and (3.10) only depend on the spatial components $\phi_{,\mathbf{m}}$, but (3.11) allows us to recursively recover the time derivatives by

$$\phi_{,\mathbf{m}+\hat{0}}(t) = \dot{\phi}_{,\mathbf{m}}(t) - \sum_{i=1}^{d} \dot{q}^i(t) \phi_{,\mathbf{m}+\hat{\imath}}(t). \tag{3.13}$$

This notation is self-consistent, because the spacetime decomposition of the relative field $(\partial_0)^n \phi_R(x)$ is

$$(\partial_0)^n \phi_R(t, \mathbf{x}) = \sum_{\mathbf{m} \in \mathbb{N}^d} \frac{1}{\mathbf{m}!} \phi_{,\mathbf{m}+n\hat{0}}(t) \mathbf{x}^{\mathbf{m}}, \tag{3.14}$$

which is precisely what we get by applying to (3.10) the operator ∂_0, defined in (3.11).

3.3 Poisson brackets

The configuration space in QJT is spanned by the Taylor coefficients $\phi_{,\mathbf{m}}$ and the observer's position q^i. Introduce canonical momenta $\pi^{,\mathbf{n}}$ and p_j, which by definition satisfy the Poisson brackets

$$\begin{aligned} [\phi_{,\mathbf{m}}(t), \pi^{,\mathbf{n}}(t)]_{PB} &= \delta^{\mathbf{n}}_{\mathbf{m}}, \\ [q^i(t), p_j(t)]_{PB} &= -\delta^i_j. \end{aligned} \tag{3.15}$$

All other equal-time brackets are assumed to vanish. In Minkowki space, vector indices are raised and lowered by means of the flat metric η_{ij}, e.g.

$$q_i(t) = \eta_{ij} q^j(t) = -q^i(t). \tag{3.16}$$

In general relativity, we instead use the metric field on the observer's trajectory, e.g.

$$\dot{q}_i(t) = g^R_{ij}(t, \mathbf{0}) \dot{q}^j(t) = g^A_{ij}(t, \mathbf{q}(t)) \dot{q}^j(t). \tag{3.17}$$

However, the upper multi-index in $\pi^{,\mathbf{m}}$ can not be lowered with the Minkowski metric in a meaningful way. Instead, the natural definition of p-jet momentum with a multi-index downstairs is

$$\pi_{,\mathbf{m}}(t) = (-)^{\mathbf{n}} \partial_{\mathbf{m}+\mathbf{n}} \delta(\mathbf{0}) \pi^{,\mathbf{n}}(t), \tag{3.18}$$

where $\delta(\mathbf{0})$ is the delta function evaluated at the origin and $\partial_{\mathbf{m}+\mathbf{n}}$ denotes the $(\mathbf{m}+\mathbf{n})$:th derivative, defined as in (3.4). This is of course a formal expression, which must be given a definite meaning. E.g., we may define the delta function as the limiting value of a family of narrowly peaked Gaussians.

The nonzero Poisson brackets in jet space are

$$\begin{aligned} [\phi_{,\mathbf{m}}(t), \pi_{,\mathbf{n}}(t)]_{PB} &= (-)^{\mathbf{n}} \partial_{\mathbf{m}+\mathbf{n}} \delta(\mathbf{0}), \\ [q^i(t), p_j(t)]_{PB} &= \delta^i_j. \end{aligned} \tag{3.19}$$

We can now define the absolute and relative field momenta by

$$\begin{aligned} \pi_A(t,\mathbf{x}) &= \sum_{\mathbf{m}\in\mathbb{N}^d} \frac{1}{\mathbf{m}!} \pi_{,\mathbf{m}}(t)(\mathbf{x}-\mathbf{q}(t))^{\mathbf{m}}, \\ \pi_R(t,\mathbf{x}) &= \sum_{\mathbf{m}\in\mathbb{N}^d} \frac{1}{\mathbf{m}!} \pi_{,\mathbf{m}}(t)\mathbf{x}^{\mathbf{m}}. \end{aligned} \tag{3.20}$$

The absolute fields satisfy the nonzero Poisson brackets

$$\begin{aligned} [\phi_A(t,\mathbf{x}), \pi_A(t,\mathbf{x}')]_{PB} &= \delta(\mathbf{x}-\mathbf{x}'), \\ [q^i(t), p_j(t)]_{PB} &= \delta^i_j, \\ [p_i(t), \phi_A(t,\mathbf{x})]_{PB} &= \partial_i \phi(t,\mathbf{x}), \\ [p_i(t), \pi_A(t,\mathbf{x})]_{PB} &= \partial_i \pi(t,\mathbf{x}). \end{aligned} \tag{3.21}$$

The absolute field and its momentum do not commute with the observer's momentum, because

$$[p_i(t), (x-q(t))^{\mathbf{m}}]_{PB} = m_i (x-q(t))^{\mathbf{m}-\hat{\imath}}. \tag{3.22}$$

In contrast, the relative field is defined by the MacLaurin series (3.10) and is independent of $q^i(t)$. It satisfies the Heisenberg algebra with nonzero brackets

$$\begin{aligned} [\phi_R(t,\mathbf{x}), \pi_R(t,\mathbf{x}')]_{PB} &= \delta(\mathbf{x}-\mathbf{x}'), \\ [q^i(t), p_j(t)]_{PB} &= \delta^i_j. \end{aligned} \tag{3.23}$$

In particular,

$$[p_i(t), \phi_R(t,\mathbf{x})]_{PB} = [p_i(t), \pi_R(t,\mathbf{x})]_{PB} = 0. \tag{3.24}$$

We now see why the jet momentum with lower multi-index must be defined as in (3.18). Computing the $[\phi_A(\mathbf{x}), \pi_A(\mathbf{x}')]$ bracket using their Taylor series defintion, we find

$$\begin{aligned}\sum_{\mathbf{m}}\sum_{\mathbf{n}}\frac{1}{\mathbf{m}!}\frac{1}{\mathbf{n}!}(-)^{\mathbf{n}}\partial_{\mathbf{m}+\mathbf{n}}\delta(\mathbf{0})(\mathbf{x}-\mathbf{q})^{\mathbf{m}}(\mathbf{x}'-\mathbf{q})^{\mathbf{n}} &= \\ = \sum_{\mathbf{r}}\frac{1}{\mathbf{r}!}\partial_{\mathbf{r}}\delta(\mathbf{0})(\mathbf{x}-\mathbf{x}')^{\mathbf{r}} &= \delta(\mathbf{x}-\mathbf{x}'),\end{aligned} \tag{3.25}$$

where the intermediate expression is the expansion of the delta function around the origin.

3.4 Dynamics

Both the fields and the observer's trajectory are dynamical degrees of freedom in QJT, and hence we need to introduce dynamics for both. The field part of the action is the same as in field theory, but we must also add terms describing the observer's dynamics and the field-observer interaction. The observer is assumed to be a point particle travelling along the trajectory $q^\mu(t)$, in accordance with the definition of absolute and relative fields in (3.1). One could in principle consider extended observers, but an irreducible observer is pointlike.

Consider a general field theory with several absolute fields $\phi^a_A(x)$, also labelled by another index a. We posit that the action is of the form $S = S_\phi + S_q$, where

$$\begin{aligned} S_\phi &= \int d^{d+1}x\, \mathcal{L}_\phi(\phi_A, \partial_\mu\phi_A) \\ &= \iint dt\, d^dx\, \mathcal{L}_\phi(\phi_A, \partial_0\phi_A, \partial_i\phi_A), \\ S_q &= \int dt\, L_q(q, \dot q, \phi_A(\mathbf{q}(t))), \end{aligned} \tag{3.26}$$

where we recall from (3.11) that $\partial_0\phi^a_A = \dot\phi^a_A$. The Lagrangian is thus of the form

$$L(t) = L_\phi(t) + L_q(t) = \int d^dx\, \mathcal{L}_\phi(t, \mathbf{x}) + L_q(t). \tag{3.27}$$

The first part has the standard field theory form, and we assume that the dynamics of the observer and the observer-field interaction is described by a term of the form S_q.

In the field part of the action, the integrand is a function of $\phi_A(t, \mathbf{x})$ and its derivatives. Such an integral can be rewritten as

$$\begin{aligned} S_\phi &= \iint dt\, d^dx\, F(\phi_A(t, \mathbf{x}), \partial_\mu \phi_A(t, \mathbf{x})) \\ &= \iint dt\, d^dx\, F(\phi_R(t, \mathbf{x} - \mathbf{q}), \partial_\mu \phi_R(t, \mathbf{x} - \mathbf{q})) \\ &= \iint dt\, d^dy\, F(\phi_R(t, \mathbf{y}), \partial_\mu \phi_R(t, \mathbf{y})), \end{aligned} \tag{3.28}$$

where $\mathbf{y} = \mathbf{x} - \mathbf{q}$, and we assume that we are free to make a linear shift in the measure, i.e. $d^dx = d^dy$. Classically, such a shift can be done, at least as long as boundary conditions are ignored. Whether this assumption is as innocent on the quantum level is less clear, since the difference between d^dx and d^dy is an operator $d^dq(t)$. On the other hand, since the integrals over x and y are equivalent classically, it is not obvious which is right choice after quantization; even if the these integrals would disagree, the y integral may well be the physically correct choice. This subtlety is ignored in the rest of this paper.

When expressed in terms of relative fields, the action becomes

$$\begin{aligned} S_\phi &= \iint dt\, d^dx\, \mathcal{L}_\phi(\phi_R, \partial_0 \phi_R, \partial_i \phi_R), \\ S_q &= \int dt\, L_q(q, \dot{q}, \phi_R(\mathbf{0})), \end{aligned} \tag{3.29}$$

where we recall from (3.11) that

$$\partial_0 \phi_R^a = \dot{\phi}_R^a - \dot{q}^i \partial_i \phi_R^a. \tag{3.30}$$

The field part S_ϕ is thus assumed to be independent of the observer's location, except implicitly through the definition of ∂_0, and the observer-field interaction is encoded in S_q. The action (3.29) leads to the canonical momenta

$$\begin{aligned} \pi_a^R(\mathbf{x}) &= \frac{\partial \mathcal{L}_\phi}{\partial \dot{\phi}_R^a(\mathbf{x})} = \frac{\partial \mathcal{L}_\phi}{\partial \partial_0 \phi_R^a(\mathbf{x})}, \\ p_i &= \frac{\partial L_q}{\partial \dot{q}^i} - \int d^dx\, \frac{\partial \mathcal{L}_\phi}{\partial \partial_0 \phi_R^a(\mathbf{x})} \partial_i \phi_R^a(\mathbf{x}) = \frac{\partial L_q}{\partial \dot{q}^i} - P_i, \end{aligned} \tag{3.31}$$

where

$$\begin{aligned} P_i &\equiv \int d^dx\, \frac{\partial \mathcal{L}_\phi}{\partial \partial_0 \phi_R^a(\mathbf{x})} \partial_i \phi_R^a(\mathbf{x}) \\ &= \int d^dx\, \pi_a^R(\mathbf{x}) \partial_i \phi_R^a(\mathbf{x}) = -\int d^dx\, \phi_R^a(\mathbf{x}) \partial_i \pi_a^R(\mathbf{x}). \end{aligned} \tag{3.32}$$

The Hamiltonian is of form

$$\begin{aligned} H &= \int d^dx\, \pi_a^R(\mathbf{x})\dot\phi_R^a(\mathbf{x}) + p_i\dot q^i - \int d^dx\, \mathcal{L}_\phi - L_q \\ &= H_\phi + H_q, \end{aligned} \tag{3.33}$$

where the field part $H_\phi(\phi_R, \pi_R)$ has the same functional form as for absolute fields. To find an explicit expression for the observer part, we assume that the equation $p_i = \frac{\partial L_q}{\partial \dot q^i}(\mathbf{q}, \dot{\mathbf{q}})$ can be inverted to yield $\dot q^i = v^i(\mathbf{q}, \mathbf{p})$. Equation (3.31) then implies that

$$\dot q^i = v^i(\mathbf{q}, \mathbf{p} + \mathbf{P}). \tag{3.34}$$

Putting this expression back into (3.33) then yields

$$\begin{aligned} H_q &= \dot q^i(p_i + P_i) - L_q \\ &= (p_i + P_i)v^i(\mathbf{q}, \mathbf{p} + \mathbf{P}) - L_q(\mathbf{q}, \mathbf{v}(\mathbf{q}, \mathbf{p} + \mathbf{P})). \end{aligned} \tag{3.35}$$

Comparing this to the corresponding analysis for absolute fields, which yield $H_q = p_i v^i(\mathbf{q}, \mathbf{p}) - L_q(\mathbf{q}, \mathbf{v}(\mathbf{q}, \mathbf{p}))$, we see that the passage from absolute to relative fields amounts to the substitutions

$$\begin{aligned} H_\phi(\phi_A, \pi_A) &\to H_\phi(\phi_R, \pi_R), \\ H_q(\mathbf{q}, \mathbf{p}, \phi_A(\mathbf{q})) &\to H_q(\mathbf{q}, \mathbf{p} + \mathbf{P}, \phi_R(\mathbf{0})). \end{aligned} \tag{3.36}$$

To summarize:

> The passage from absolute to relative fields is equivalent to the substitution $p_i \to p_i + P_i$ in the observer part of the Hamiltonian.

The relative Hamiltonian (3.36) leads to the following Hamilton's equation:

$$\begin{aligned} \dot\phi_R^a(\mathbf{x}) &= \frac{\delta H_\phi}{\delta \pi_a^R(\mathbf{x})} + \frac{\partial H_q}{\partial p_i}\partial_i\phi_R^a(\mathbf{x}), \\ \dot\pi_a^R(\mathbf{x}) &= -\frac{\delta H_\phi}{\delta \phi_R^a(\mathbf{x})} + \frac{\partial H_q}{\partial p_i}\partial_i\pi_a^R(\mathbf{x}) - \frac{\delta H_q}{\delta \phi_R^a(\mathbf{x})}\delta(\mathbf{x}), \\ \dot q^i &= \frac{\partial H_q}{\partial p_i}, \\ \dot p_i &= -\frac{\partial H_q}{\partial q^i}. \end{aligned} \tag{3.37}$$

Combining the definition of ∂_0 in (3.11) and the evolution equation for q^i, we get

$$\partial_0 \phi_R^a(\mathbf{x}) = \dot{\phi}_R^a(\mathbf{x}) - \frac{\partial H_q}{\partial p_i} \partial_i \phi_R^a(\mathbf{x}), \tag{3.38}$$

etc. The first Hamilton's equations can thus be written in the form

$$\begin{aligned} \partial_0 \phi_R^a(\mathbf{x}) &= \frac{\delta H_\phi}{\delta \pi_a^R(\mathbf{x})}, \\ \partial_0 \pi_a^R(\mathbf{x}) &= -\frac{\delta H_\phi}{\delta \phi_R^a(\mathbf{x})} - \frac{\delta H_q}{\delta \phi_R^a(\mathbf{x})} \delta(\mathbf{x}). \end{aligned} \tag{3.39}$$

Apart from the last term, which encodes the interaction between the fields and the observer, this is of the familiar form.

3.5 Quantization

A model with relative fields can be canonically quantized as usual. Replace the Poisson brackets (3.23) with commutators and represent the Heisenberg algebra on a Hilbert space. In the very simple case that the observer does not interact with the fields, the Hilbert space becomes the tensor product $\mathcal{H}_{tot} = \mathcal{H}_{field} \otimes \mathcal{H}_{obs}$. Let $|\mathbf{k}\rangle$ and $|\mathbf{u}\rangle$ be eigenstates of H_ϕ and H_q, respectively, with eigenvalues $E_q(\mathbf{k})$ and $E_q(\mathbf{p}(\mathbf{u}))$, respectively. Moreover, assume for simplicity that $|\mathbf{u}\rangle$ and $|\mathbf{k}\rangle$ are eigenstates of p_i and P_i, respectively, *viz.*

$$p_i |\mathbf{u}\rangle = p_i(\mathbf{u}) |\mathbf{u}\rangle, \qquad P_i |\mathbf{k}\rangle = k_i |\mathbf{k}\rangle. \tag{3.40}$$

Under these assumptions, the product state $|\mathbf{k}\rangle \otimes |\mathbf{u}\rangle$ is an eigenstate of the total Hamiltonian, and the eigenvalue is

$$E(\mathbf{k}, \mathbf{u}) = E_\phi(\mathbf{k}) + E_q(\mathbf{p}(\mathbf{u}) + \mathbf{k}). \tag{3.41}$$

We deal with quantization in more detail in the examples below.

3.6 Observer versus frame dependence

It should be emphasized that observer dependence does not mean that we work in the observer's rest frame. To the contrary, since we label spacetime points by their GPS coordinates x^μ, we work in the frame of the GPS satellites. That we do not work in the observer's rest frame is easy to see, because the observer's velocity $\dot{q}^i(t) = dq^i/dq^0 \neq 0$. E.g., if the observer Hamiltonian with absolute fields is $H_q = \sqrt{M^2 + \mathbf{p}^2}$, the analogous relative quantity is $H_q = \sqrt{M^2 + (\mathbf{p} + \mathbf{P})^2}$, and not $H_q = M$ which would be the case in the observer's rest frame.

4 Free scalar field

4.1 Action

The action for a self-interacting scalar field reads $S = S_\phi + S_q$, where

$$\begin{aligned} S_\phi &= \int d^{d+1}x \, \big(\frac{1}{2}\partial_\mu\phi_A\partial^\mu\phi_A - V(\phi_A)\big), \\ S_q &= -M\int dt\, \sqrt{\dot{q}^\mu\dot{q}_\mu}. \end{aligned} \tag{4.1}$$

In particular, if the scalar field is free and has mass ω, the potential is $V(\phi) = 1/2\,\omega^2\phi^2$; we denote the mass by ω rather than m to avoid confusion with multi-indices. The field part of the action becomes

$$S_\phi = \frac{1}{2}\int d^{d+1}x\, \big((\partial_0\phi_A)^2 - (\nabla\phi_A)^2 - \omega^2\phi_A^2\big). \tag{4.2}$$

We now introduce relative fields and eliminate reparametrization freedom by (3.7). The action becomes

$$\begin{aligned} S_\phi &= \frac{1}{2}\iint dt\, d^dx\, \big((\partial_0\phi_R)^2 - (\nabla\phi_R)^2 - \omega^2\phi_R^2\big), \\ S_q &= -M\int dt\,\sqrt{1-\dot{\mathbf{q}}^2}, \end{aligned} \tag{4.3}$$

where

$$\partial_0\phi_R = \dot\phi_R + \dot{\mathbf{q}}\cdot\nabla\phi_R \equiv \dot\phi_R + \dot{q}_i\partial_i\phi_R. \tag{4.4}$$

Note the change of sign compared to (3.11), due to the contraction of two lower indices.

4.2 Equations of motion

The Lagrangian is of the form (3.27), and the Euler-Lagrange equations

$$\begin{aligned} \frac{\delta S}{\delta\phi_R} &= -\frac{d}{dt}\frac{\partial\mathcal{L}_\phi}{\partial\dot\phi_R} - \partial_i\frac{\partial\mathcal{L}_\phi}{\partial(\partial_i\phi_R)} + \frac{\partial\mathcal{L}_\phi}{\partial\phi_R} + \frac{\partial L_q}{\partial\phi_R}\delta(\mathbf{x}) = 0, \\ \frac{\delta S}{\delta q_i} &= -\frac{d}{dt}\frac{\partial L}{\partial\dot{q}_i} + \frac{\partial L}{\partial q_i} = 0, \end{aligned} \tag{4.5}$$

become

$$-\partial_0^2\phi_R(t,\mathbf{x}) + \nabla^2\phi_R(t,\mathbf{x}) - \omega^2\phi_R(t,\mathbf{x}) = 0, \tag{4.6}$$

$$\frac{d}{dt}\big(M\gamma(\dot{\mathbf{q}}(t))\dot{q}_i(t) + P_i(t)\big) = 0. \tag{4.7}$$

Here we use the standard notation

$$\gamma(\mathbf{u}) \equiv \frac{1}{\sqrt{1-\mathbf{u}^2}}, \tag{4.8}$$

and introduce the operator which measures the field momentum:

$$P_i(t) \equiv \int d^dx\, \partial_0\phi_R(t,\mathbf{x})\, \partial_i\phi_R(t,\mathbf{x}). \tag{4.9}$$

The solution of the Euler-Lagrange equation for ϕ_R is straightforward. We know that the corresponding absolute field solution is a sum over plane waves

$$\phi_A(t,\mathbf{x}) = \exp(ik_0t - i\mathbf{k}\cdot\mathbf{x}), \tag{4.10}$$

with energy $k_0 = \pm\omega_{\mathbf{k}}$, where

$$\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + \omega^2}. \tag{4.11}$$

The relative field solution to (4.6) is thus

$$\phi_R(t,\mathbf{x}) = \exp(ik_0t - i\mathbf{k}\cdot(\mathbf{x} + \mathbf{q}(t))), \tag{4.12}$$

with the same dispersion relation. We can now evaluate

$$P_j(t) = -k_0k_j \exp(2ik_0t - 2i\mathbf{k}\cdot\mathbf{q}(t)) \int d^dx\, exp(-2i\mathbf{k}\cdot\mathbf{x}). \tag{4.13}$$

The integral is proportional to $\delta(\mathbf{k})$, and since $k_j\delta(\mathbf{k}) = 0$, we find that

$$P_j(t) = 0. \tag{4.14}$$

This result has only been checked for free fields, but it seems likely to hold generically. The solution to the evolution equation (4.7) is

$$M\gamma(\dot{\mathbf{q}}(t))\dot{q}_j(t) = p_j, \tag{4.15}$$

for some constants p_j. Defining velocities u_j by $p_j = M\gamma(\mathbf{u})u_j$, we finally find that the observer moves along the straight line

$$q_j(t) = u_jt + s_j, \tag{4.16}$$

for some constants u_j and s_j. This is the expected result; since the field and the observer do not interact, they evolve as a free field and as a free particle, respectively.

We observed in the previous section that in the Hamiltonian formulation, the full effect of working with relative fields is to shift the observer's momentum $\mathbf{p} \to \mathbf{p} - \mathbf{P}$. Equation (4.14) then asserts that for classical solutions, $\mathbf{P} = 0$, so relative fields do not give anything new classically; QJT only differs significantly from QFT on the quantum level. This was expected, since the only difference between QJT and QFT is that the former takes the observer's quantum dynamics into account.

4.3 Hamiltonian

The relative canonical momenta read

$$\begin{aligned} \pi_R(t, \mathbf{x}) &= \frac{\partial \mathcal{L}_\phi}{\partial \dot{\phi}_R} = \partial_0 \phi_R(t, \mathbf{x}), \\ p_j(t) &= \frac{\partial L_q}{\partial \dot{q}_j} + \frac{\partial L_\phi}{\partial \dot{q}_j} = M\gamma(\dot{\mathbf{q}}(t))\dot{q}_j(t) + P_j(t), \end{aligned} \tag{4.17}$$

where $\gamma(\dot{\mathbf{q}})$ was defined in (4.8), and the definition of $P_i(t)$ in (4.9) becomes

$$\begin{aligned} P_i(t) &= \int d^dx\, \pi_R(t, \mathbf{x})\, \partial_i \phi_R(t, \mathbf{x}) \\ &= -\int d^dx\, \partial_i \pi_R(t, \mathbf{x})\, \phi_R(t, \mathbf{x}). \end{aligned} \tag{4.18}$$

The Hamiltonian is

$$H = \int d^dx\, \pi_R \dot{\phi}_R + p_j \dot{q}_j - L = H_\phi + H_q, \tag{4.19}$$

where

$$\begin{aligned} H_\phi &= \frac{1}{2} \int d^dx\, \left(\pi_R^2 + (\nabla \phi_R)^2 + \omega^2 \phi_R^2\right), \\ H_q &= \sqrt{M^2 + (\mathbf{p} - \mathbf{P})^2}, \end{aligned} \tag{4.20}$$

and $(\mathbf{p} - \mathbf{P})^2 = (p_i - P_i)(p_i - P_i)$.

Hamilton's equations read

$$\begin{aligned} \dot{\phi}_R(\mathbf{x}) &= \frac{\delta H}{\delta \pi(\mathbf{x})} = \pi(\mathbf{x}) - \frac{p_i - P_i}{\sqrt{M^2 + (\mathbf{p} - \mathbf{P})^2}}\, \partial_i \phi_R(\mathbf{x}), \\ \dot{\pi}_R(\mathbf{x}) &= -\frac{\delta H}{\delta \phi(\mathbf{x})} = \nabla^2 \phi_R(\mathbf{x}) - \omega^2 \phi_R(\mathbf{x}) - \frac{p_i - P_i}{\sqrt{M^2 + (\mathbf{p} - \mathbf{P})^2}}\, \partial_i \pi_R(\mathbf{x}), \\ \dot{q}_i &= \frac{\partial H}{\partial p_i} = \frac{p_i - P_i}{\sqrt{M^2 + (\mathbf{p} - \mathbf{P})^2}}, \\ \dot{p}_j &= -\frac{\partial H}{\partial q_i} = 0. \end{aligned} \tag{4.21}$$

Using (4.4) in the form

$$\partial_0\phi_R = \dot{\phi}_R + \frac{(p_i - P_i)}{\sqrt{M^2 + (\mathbf{p} - \mathbf{P})^2}}\,\partial_i\phi_R, \tag{4.22}$$

Hamilton's equations can be rewritten as

$$\begin{aligned}
\partial_0\phi_R(\mathbf{x}) &= \pi_R(\mathbf{x}),\\
\partial_0\pi_R(\mathbf{x}) &= \nabla^2\phi_R(\mathbf{x}) - \omega^2\phi_R(\mathbf{x}),\\
M\gamma(\dot{\mathbf{q}})\dot{q}^i &= p_i - P_i,\\
\dot{p}_j &= 0.
\end{aligned} \tag{4.23}$$

The solution is of course still given by (4.12) and (4.16).

4.4 Quantization

Now we quantize the theory by replacing Poisson brackets by commutators. The only nonzero brackets in the Heisenberg algebra are

$$\begin{aligned}
[\phi_R(t,\mathbf{x}), \pi_R(t,\mathbf{x}')] &= i\delta(\mathbf{x} - \mathbf{x}'),\\
[q_i(t), p_j(t)] &= i\delta_{ij}.
\end{aligned} \tag{4.24}$$

We use units such that $\hbar = 1$, but occationally reinsert factors of $\hbar$ when needed for clarity. After a Fourier transformation in space only, the modes satisfy

$$[\phi_R(t,\mathbf{k}), \pi_R(t,\mathbf{k}')] = i\delta(\mathbf{k} + \mathbf{k}'). \tag{4.25}$$

Define creation and annihilation operators by

$$\begin{aligned}
a_{\mathbf{k}} &= \frac{1}{\sqrt{2\omega_{\mathbf{k}}}}(\pi_R(\mathbf{k}) - i\omega_{\mathbf{k}}\phi_R(\mathbf{k})),\\
a^\dagger_{\mathbf{k}} &= \frac{1}{\sqrt{2\omega_{\mathbf{k}}}}(\pi_R(\mathbf{k}) + i\omega_{\mathbf{k}}\phi_R(\mathbf{k})).
\end{aligned} \tag{4.26}$$

The complete set of nonzero commutation relations is thus

$$\begin{aligned}
[a_{\mathbf{k}}(t), a^\dagger_{\mathbf{k}'}(t)] &= \delta(\mathbf{k} + \mathbf{k}'),\\
[q_i(t), p_j(t)] &= i\delta_{ij},
\end{aligned} \tag{4.27}$$

where we also remember that the system depends on the observer's position and momentum. Introduce the number operator

$$N = \int d^dk\, a^\dagger_{\mathbf{k}} a_{-\mathbf{k}}. \tag{4.28}$$

The extra term P_j, defined in (4.18), takes the form

$$P_j = i \int d^d k \, k_j \pi_R(-\mathbf{k}) \phi(\mathbf{k}) = \int d^d k \, k_j a^\dagger_{\mathbf{k}} a_{-\mathbf{k}}. \tag{4.29}$$

N and P_j satisfy

$$\begin{aligned} [N, a_{\mathbf{k}}] &= -a_{\mathbf{k}}, \qquad & [N, a^\dagger_{\mathbf{k}}] &= a^\dagger_{\mathbf{k}}, \\ [P_j, a_{\mathbf{k}}] &= k_j a_{\mathbf{k}}, \qquad & [P_j, a^\dagger_{\mathbf{k}}] &= k_j a^\dagger_{\mathbf{k}}. \end{aligned} \tag{4.30}$$

N is thus the number operator for quanta, and P_j the operator that counts the momentum of the field quanta.

The field part of the Hamiltonian acquires the familiar form

$$H_\phi = \int d^d k \, \omega_{\mathbf{k}} a^\dagger_{\mathbf{k}} a_{-\mathbf{k}}, \tag{4.31}$$

and the total Hamiltonian is $H = H_\phi + H_q$, where as before

$$H_q = \sqrt{M^2 + (\mathbf{p} - \mathbf{P})^2}. \tag{4.32}$$

We note that $[H_\phi, H_q] = 0$, and that N, p_i and P_i commute with both H_ϕ and H_q and among themselves. We can therefore diagonalize H_ϕ, H_q, N, p_i and P_i simultaneously. The Fock space is spanned by states with N quanta with energy H_ϕ and momentum P_i, and the observer has energy H_q and momentum p_i. In particular, the Fock vacuum is a tensor product fully characterized by the observer's velocity $\mathbf{u}$:

$$\big|0; \mathbf{u}\big\rangle = \big|0\big\rangle \otimes \big|\mathbf{u}\big\rangle. \tag{4.33}$$

The general n-quanta state,

$$\big|\{\mathbf{k}\}; \mathbf{u}\big\rangle \equiv \big|\mathbf{k}_1, ..., \mathbf{k}_n; \mathbf{u}\big\rangle = a^\dagger_{\mathbf{k}_1} ... a^\dagger_{\mathbf{k}_n} \big|0\big\rangle \otimes \big|\mathbf{u}\big\rangle, \tag{4.34}$$

is an unnormalized eigenstate with the following eigenvalues:

$$\begin{aligned} N \big|\{\mathbf{k}\}; \mathbf{u}\big\rangle &= n \big|\{\mathbf{k}\}; \mathbf{u}\big\rangle, \\ P_j \big|\{\mathbf{k}\}; \mathbf{u}\big\rangle &= \sum_{\ell=1}^{n} k_{\ell_j} \big|\{\mathbf{k}\}; \mathbf{u}\big\rangle, \\ p_j \big|\{\mathbf{k}\}; \mathbf{u}\big\rangle &= M\gamma u_j \big|\{\mathbf{k}\}; \mathbf{u}\big\rangle, \\ H \big|\{\mathbf{k}\}; \mathbf{u}\big\rangle &= E(\{\mathbf{k}\}; \mathbf{u}) \big|\{\mathbf{k}\}; \mathbf{u}\big\rangle, \end{aligned} \tag{4.35}$$

where $\gamma = \gamma(\mathbf{u})$ is given by (4.8). The energy of the n-quanta state is

$$E(\{\mathbf{k}\};\mathbf{u}) = \sum_{\ell=1}^{n} \omega_{\mathbf{k}_\ell} + \sqrt{M^2 + (M\gamma\mathbf{u} - \sum_{\ell=1}^{n} \mathbf{k}_\ell)^2}. \tag{4.36}$$

In particular, the energy of the ground state is

$$E(\mathbf{u}) = \sqrt{M^2 + (M\gamma\mathbf{u})^2} = M\gamma, \tag{4.37}$$

which is recognized as the energy of a particle of mass M and velocity $\mathbf{u}$, which is how we have modeled the observer. The energy of a one-quantum state is

$$E(\mathbf{k};\mathbf{u}) = \omega_{\mathbf{k}} + \sqrt{M^2 + (M\gamma\mathbf{u} - \mathbf{k})^2}. \tag{4.38}$$

If we set $\mathbf{u} = \mathbf{0}$, the energy of the one-quantum state reduces to

$$E(\mathbf{k};\mathbf{0}) = \omega_{\mathbf{k}} + \sqrt{M^2 + \mathbf{k}^2}. \tag{4.39}$$

Thus it appears as the observer's mass is $\mathbf{k}$-dependent, and equal to $M(\mathbf{k}) = \sqrt{M^2 + \mathbf{k}^2}$. On the other hand, in a situation where the observer's momentum equals the momentum of the quantum, i.e. $M\gamma\mathbf{u} = \mathbf{k}$, the energy reaches its minimum value

$$E(\mathbf{k};\mathbf{u}) = \omega_{\mathbf{k}} + M. \tag{4.40}$$

These formulas also apply to the multi-quanta state (4.36), provided that we interpret $\mathbf{k}$ as the total momentum of all quanta, i.e. $\mathbf{k} = \sum_{\ell=1}^{n} \mathbf{k}_\ell$.

4.5 Non-relativistic limit

Let us now specialize to the case that observer is much heavier than the energy of the quanta, i.e. the limit $M \to \infty$. The single-quantum energy (4.38) becomes

$$E(\mathbf{k};\mathbf{u}) \approx M\gamma + \omega_{\mathbf{k}} - \mathbf{u}\cdot\mathbf{k} + \frac{1}{2M\gamma}(\mathbf{k}^2 - 2(\mathbf{u}\cdot\mathbf{k})^2) + O\left(\frac{|\mathbf{k}|^3}{M^2}\right). \tag{4.41}$$

The first term is simply the relativistic energy (4.37) of the point-like observer. The next two terms are proportional to $\hbar$ and independent of the observer's mass. They express that an observer that moves with velocity $\mathbf{u}$

experiences a Doppler shift. Relative to the moving observer, the frequency of a quantum with wave vector $\mathbf{k}$ is shifted to

$$\omega_{\mathbf{k}} \longrightarrow \omega_{\mathbf{k}} - \mathbf{u} \cdot \mathbf{k}. \tag{4.42}$$

The final term is a genuinely new effect which is due to the observer dependence of relative fields. It asserts that in addition to the Doppler shift, the frequency of the one-quantum state acquires an extra shift

$$\frac{1}{2M\gamma}(\mathbf{k}^2 - 2(\mathbf{u} \cdot \mathbf{k})^2). \tag{4.43}$$

Note that this effect is present even if the observer is not moving relative to the global origin. When $\mathbf{u} = 0$, the energy of a single-quantum state is $E(\mathbf{k}; \mathbf{0}) = M + \epsilon_{\mathbf{k}}$, where

$$\epsilon_{\mathbf{k}} \approx \omega_{\mathbf{k}} + \frac{|\mathbf{k}|^2}{2M}. \tag{4.44}$$

The energy of the quanta is not additive. The total energy of the observer and quanta with wave-vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ is

$$E(\mathbf{k}_1, \mathbf{k}_2; \mathbf{0}) \approx M + \epsilon_{\mathbf{k}_1} + \epsilon_{\mathbf{k}_2} + \frac{1}{M}\mathbf{k}_1 \cdot \mathbf{k}_2. \tag{4.45}$$

The last interference term appears as an interaction between the two quanta, and is a genuine QJT effect; it disappears in the QFT limit $M \to \infty$.

4.6 Other observer states

So far we assumed that the observer is in a velocity eigenstate $\big|\mathbf{u}\big\rangle$. Since the observer obeys the rules of quantum mechanics, this means that its position is entirely unknown. A general observer state $\big|f\big\rangle$ is some linear superposition of velocity eigenstates, *viz.*

$$\big|f\big\rangle = \int d^d u\, f(\mathbf{u})\, \big|\mathbf{u}\big\rangle, \tag{4.46}$$

and the Fock space can be constructed by applying creation operators to the vacuum $\big|f\big\rangle$. In particular, a position eigenstate is the linear superposition

$$\big|\mathbf{x}\big\rangle = \int d^d p(\mathbf{u})\, \mathrm{e}^{i\mathbf{p}(\mathbf{u})\cdot\mathbf{x}}\, \big|\mathbf{u}\big\rangle, \tag{4.47}$$

where $\mathbf{p}(\mathbf{u}) = M\gamma(\mathbf{u})\mathbf{u}$. When acting on a state $\big|\{\mathbf{k}\};\mathbf{x}\big\rangle$ with n quanta with wave-vectors $\mathbf{k}_\ell$, the Hamiltonian takes the form $H = H_\phi + H_q$, where

$$\begin{aligned} H_\phi &= \sum_{\ell=1}^{n} \omega_{\mathbf{k}_\ell}, \\ H_q &= \sqrt{M^2 + (i\nabla + \sum_{\ell=1}^{n} \mathbf{k}_\ell)^2}. \end{aligned} \tag{4.48}$$

We only consider velocity eigenstates in this article.

5 The free electromagnetic field

5.1 Action and Hamiltonian

We next turn to describe electromagnetism in terms of relative fields. Here we encounter two new phenomena: gauge symmetry and interaction between the observer and the fields. To lighten the notation, the subscript R is suppressed, keeping in mind that all fields are relative. The action consists of three terms, which describe the electromagnetic field itself, the observer's trajectory, and the interaction between the field and the observer. We assume that the observer is a charged particle with charge e. The presence of an explicit field-observer interaction is the main novelty in this section.

The action reads $S = S_A + S_q + S_{qA}$, where

$$\begin{aligned} S_A &= -\frac{1}{4}\iint dt\, d^dx\, F^{\mu\nu}(t,\mathbf{x})F_{\mu\nu}(t,\mathbf{x}) \\ &= \iint dt\, d^dx\, \big(\frac{1}{2}F_{0i}(t,\mathbf{x})F_{0i}(t,\mathbf{x}) - \frac{1}{4}F_{ij}(t,\mathbf{x})F_{ij}(t,\mathbf{x})\big), \\ S_q &= -M\int dt\, \sqrt{1-\dot{\mathbf{q}}^2(t)}, \\ S_{qA} &= e\int dt\, \dot{q}^\mu(t)A_\mu(t,\mathbf{0}) = e\int dt\, (A_0(t,\mathbf{0}) - \dot{q}_i(t)A_i(t,\mathbf{0})). \end{aligned} \tag{5.1}$$

As usual, the field strength is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{5.2}$$

In particular,

$$\begin{aligned} F_{0\mu}(t,\mathbf{x}) &= \partial_0 A_\mu(t,\mathbf{x}) - \partial_\mu A_0(t,\mathbf{x}) \\ &= \dot{A}_\mu(t,\mathbf{x}) + \dot{q}_j\partial_j A_\mu(t,\mathbf{x}) - \partial_\mu A_0(t,\mathbf{x}). \end{aligned} \tag{5.3}$$

The canonical momenta are

$$\begin{aligned}
E_\mu(\mathbf{x}) &\equiv \frac{\partial \mathcal{L}}{\partial \dot{A}_\mu(\mathbf{x})} = F_{0\mu}(\mathbf{x}), \\
p_i &\equiv \frac{\partial L}{\partial \dot{q}_i} = M\gamma(\dot{\mathbf{q}})\dot{q}_i + P_i - eA_i(\mathbf{0}),
\end{aligned} \tag{5.4}$$

where

$$\begin{aligned}
P_i &= \int d^dx\, \partial_i A_j(\mathbf{x}) F_{0j}(\mathbf{x}) \\
&= \int d^dx\, \partial_i A_j(\mathbf{x}) E_j(\mathbf{x}) = -\int d^dx\, A_j(\mathbf{x}) \partial_i E_j(\mathbf{x}).
\end{aligned} \tag{5.5}$$

We postulate the nonzero canonical commutators

$$\begin{aligned}
[A_\mu(\mathbf{x}), E_\nu(\mathbf{x}')] &= -i\eta_{\mu\nu}\delta(\mathbf{x}-\mathbf{x}'), \\
[q_i, p_j] &= i\delta_{ij}.
\end{aligned} \tag{5.6}$$

Because of the primary constraint

$$E_0(\mathbf{x}) \approx 0, \tag{5.7}$$

the Hamiltonian $H = H_A + H_q$ depends on an arbitrary Lagrange multiplier $u_1(\mathbf{x})$:

$$\begin{aligned}
H &= \int d^dx\, \dot{A}_\mu(\mathbf{x}) E^\mu(\mathbf{x}) + \dot{q}_i p_i - L + \int d^dx\, u_1(\mathbf{x}) E_0(\mathbf{x}), \\
H_A &= \int d^dx\, \big(\frac{1}{2} E_i E_i + \frac{1}{4} F_{ij} F_{ij} + E_i \partial_i A_0 + u_1 E_0\big), \\
H_q &= \sqrt{M^2 + (\mathbf{p} - \mathbf{P} + e\mathbf{A}(\mathbf{0}))^2},
\end{aligned} \tag{5.8}$$

Hamilton's equations read

$$\begin{aligned}
\partial_0 A_i(\mathbf{x}) &= E_i(\mathbf{x}) + \partial_i A_0(\mathbf{x}), \\
\partial_0 E_i(\mathbf{x}) &= \partial_j F_{ji}(\mathbf{x}) - e\dot{q}_i \delta(\mathbf{x}), \\
\dot{A}_0(\mathbf{x}) &= -u_1(\mathbf{x}), \\
\dot{E}_0(\mathbf{x}) &= -\partial_i E_i(\mathbf{x}) + e\delta(\mathbf{x}), \\
\dot{q}_i &= \frac{p_i - P_i + eA_i(\mathbf{0})}{\sqrt{M^2 + (\mathbf{p} - \mathbf{P} + e\mathbf{A}(\mathbf{0}))^2}}, \\
\dot{p}_i &= 0,
\end{aligned} \tag{5.9}$$

where the action of ∂_0 on relative fields is still defined by (4.4). Time evolution of the constraint (5.7) gives rise to the secondary constraint (Gauss' law)

$$J(\mathbf{x}) \equiv \partial_i E_i(\mathbf{x}) - e\delta(\mathbf{x}) \approx 0. \tag{5.10}$$

Gauss' law commutes with the Hamiltonian, since $\partial_0\delta(\mathbf{x}) = \dot{q}_i\partial_i\delta(\mathbf{x})$, and hence there are no further constraints. Gauss' law allows us to add the term $H_2 = \int d^dx\, u_2(\mathbf{x})J(\mathbf{x})$ to the Hamiltonian. This modifies the time evolution for A_i into

$$\partial_0 A_i(\mathbf{x}) = E_i(\mathbf{x}) + \partial_i A_0(\mathbf{x}) - \partial_i u_2(\mathbf{x}). \tag{5.11}$$

There are several elegant methods to deal with constrained Hamiltonian systems. Since our interest is to exhibit the effects of observer dependence, we simply solve the constraint by imposing the gauge fixing conditions

$$A_0(\mathbf{x}) = \partial_i A_i(\mathbf{x}) = 0, \tag{5.12}$$

and replacing Poisson brackets by Dirac brackets. The physical degrees of freedom transverse fields $A_i^T(\mathbf{x})$ and $E_i^T(\mathbf{x})$. The Hamiltonian consists of two terms, $H = H_A + H_q$, where the field part takes the familiar form in four dimensions:

$$H_A = \frac{1}{2}\int d^dx\, \big(E_i^T(\mathbf{x})E_i^T(\mathbf{x}) + B_i^T(\mathbf{x})B_i^T(\mathbf{x})\big), \tag{5.13}$$

where the magnetic field is $B_i = \frac{1}{2}\epsilon_{ijk}F_{jk} = \epsilon_{ijk}\partial_j A_k$.

5.2 Fourier space

The fundamental brackets in Fourier space,

$$[A_i(\mathbf{k}), E_j(\mathbf{k}')] = i\delta_{ij}\delta(\mathbf{k}+\mathbf{k}'), \tag{5.14}$$

imply that

$$[E_i(\mathbf{k}), B_j(\mathbf{k}')] = -\epsilon_{ij\ell}k_\ell\delta(\mathbf{k}+\mathbf{k}'), \tag{5.15}$$

where $B_i(\mathbf{k}) = i\epsilon_{ij\ell}k_j A_\ell(\mathbf{k})$ are the Fourier components of the magnetic field. It is useful to consider the dual magnetic field

$$\tilde{B}_i(\mathbf{k}) = i\epsilon_{ij\ell}\frac{k_j}{|\mathbf{k}|}B_\ell(\mathbf{k}) = |\mathbf{k}|\Delta_{ij}(\mathbf{k})A_j(\mathbf{k}), \tag{5.16}$$

where

$$\Delta_{ij}(\mathbf{k}) = \Delta_{ji}(\mathbf{k}) = \delta_{ij} - \frac{k_i k_j}{|\mathbf{k}|^2} \tag{5.17}$$

satisfies $k_j \Delta_{ij}(\mathbf{k}) = 0$. Introduce the oscillators

$$\begin{aligned} a_i(\mathbf{k}) &= \frac{1}{\sqrt{2|\mathbf{k}|}}(E_i(\mathbf{k}) - i\tilde{B}_i(k)), \\ a_i^\dagger(\mathbf{k}) &= \frac{1}{\sqrt{2|\mathbf{k}|}}(E_i(\mathbf{k}) + i\tilde{B}_i(k)), \end{aligned} \tag{5.18}$$

which satisfy the CCR

$$[a_i(\mathbf{k}), a_j^\dagger(\mathbf{k}')] = \Delta_{ij}(\mathbf{k})\delta(\mathbf{k}+\mathbf{k}'). \tag{5.19}$$

The Gauss law constraint (5.10) takes the form

$$\begin{aligned} J(\mathbf{k}) &= k_i E_i(\mathbf{k}) - e \qquad (5.20) \\ &= i\sqrt{\frac{|\mathbf{k}|}{2}}\Big(k_i a_i(\mathbf{k}) + k_i a_i^\dagger(\mathbf{k})\Big) - e \approx 0. \end{aligned}$$

It is compatible with the brackets (5.19), because the observer's charge e commutes with $a_i(\mathbf{k})$ and $a_i^\dagger(\mathbf{k})$. Of the d pairs of oscillators, only $d-1$ are independent. We therefore introduce the standard polarization vectors $\epsilon_{i\alpha}(\mathbf{k})$, where α, β run over the $d-1$ transverse directions. The following relations hold:

$$\epsilon_{i\alpha}(\mathbf{k})\epsilon_{i\beta}(-\mathbf{k}) = \delta_{\alpha\beta}, \qquad k_i\epsilon_{i\alpha}(\mathbf{k}) = 0. \tag{5.21}$$

The transverse oscillators,

$$a_\alpha(\mathbf{k}) = \epsilon_{i\alpha}(-\mathbf{k})a_i(\mathbf{k}), \qquad a_\alpha^\dagger(\mathbf{k}) = \epsilon_{i\alpha}(-\mathbf{k})a_i^\dagger(\mathbf{k}), \tag{5.22}$$

satisfy a non-degenerate Heisenberg algebra,

$$[a_\alpha(\mathbf{k}), a_\beta^\dagger(\mathbf{k}')] = \delta_{\alpha\beta}\delta(\mathbf{k}+\mathbf{k}'). \tag{5.23}$$

The field part of the Hamiltonian H_A can now be written as a sum of $d-1$ independent harmonic oscillators,

$$H_A = \sum_{\alpha=1}^{d-1} \int d^dk\, |\mathbf{k}| a_\alpha^\dagger(\mathbf{k}) a_\alpha(-\mathbf{k}). \tag{5.24}$$

Note that this property is not destroyed by the presence of the observer's charge in (5.20). The observer part is more interesting, since it contains the interaction term

$$H_q = \sqrt{M^2 + (\mathbf{p} - \mathbf{P} + e\mathbf{A}(\mathbf{0}))^2}. \tag{5.25}$$

To lowest order in the charge, we can write the Hamiltonian as

$$H_q = H_q^0 + eH_q^1 + O(e^2), \tag{5.26}$$

where

$$\begin{aligned} H_q^0 &= \sqrt{M^2 + (\mathbf{p} - \mathbf{P})^2}, \\ H_q^1 &= \frac{A_i(\mathbf{0})(p_i - P_i)}{\sqrt{M^2 + (\mathbf{p} - \mathbf{P})^2}}. \end{aligned} \tag{5.27}$$

The H_q^1 term suffers from an ordering ambiguity, because $A_i(\mathbf{0})$ and P_i do not commute. Since

$$\begin{aligned} A_\alpha(\mathbf{x}) &= \int d^dk\, \mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}} \epsilon_{i\alpha}(\mathbf{k}) A_i(\mathbf{k}) \\ &= i \int \frac{d^dk}{\sqrt{2|\mathbf{k}|}} \mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}} \epsilon_{i\alpha}(\mathbf{k})(a_\alpha(\mathbf{k}) - a_\alpha^\dagger(\mathbf{k})), \end{aligned} \tag{5.28}$$

we define H_q^1 as the normal-ordered expression

$$H_q^1 = i \int \frac{d^dk}{\sqrt{2|\mathbf{k}|}} \epsilon_{i\alpha}(\mathbf{k}) (\frac{p_i - P_i}{H_q^0} a_\alpha(\mathbf{k}) - a_\alpha^\dagger(\mathbf{k}) \frac{p_i - P_i}{H_q^0}). \tag{5.29}$$

5.3 Quantization

Let $\big|\mathbf{u}\big\rangle$ be the vacuum state in the presence of an observer moving with velocity $\mathbf{u}$, and let

$$\big|(\mathbf{k}_1, \alpha_1), ...(\mathbf{k}_n, \alpha_n); \mathbf{u}\big\rangle = a_{\alpha_1}^\dagger(\mathbf{k}_1) ... a_{\alpha_n}^\dagger(\mathbf{k}_n) \big|\mathbf{u}\big\rangle \tag{5.30}$$

denote an n-photon state built over it. The photons are characterized by their momentum $\mathbf{k}_\ell$ and polarization α_ℓ. In particular, we will consider the one-photon state $\big|(\mathbf{k}, \alpha); \mathbf{u}\big\rangle$ and the two-photon state $\big|(\mathbf{k}, \alpha), (\mathbf{k}', \beta); \mathbf{u}\big\rangle$.

Introduce the dual states

$$\big\langle(\mathbf{k}_1, \alpha_1), ...(\mathbf{k}_n, \alpha_n); \mathbf{u}\big| = \big\langle\mathbf{u}\big| a_{\alpha_n}(-\mathbf{k}_n) ... a_{\alpha_1}(-\mathbf{k}_1). \tag{5.31}$$

The inner product defined by $\big\langle \mathbf{u} \big| \mathbf{u}' \big\rangle = \delta(\mathbf{u}-\mathbf{u}')$ is in general not normalized. However, one-photon states are normalized,

$$\big\langle (\mathbf{k},\alpha); \mathbf{u} \big| (\mathbf{k}',\beta); \mathbf{u}' \big\rangle = \delta(\mathbf{k}-\mathbf{k}')\delta_{\alpha\beta}\delta(\mathbf{u}-\mathbf{u}'), \tag{5.32}$$

as are multi-photon states where are photons are different.

The vacuum satisfies

$$p_i \big|\mathbf{u}\big\rangle = M\gamma(\mathbf{u})u_i \big|\mathbf{u}\big\rangle, \qquad P_i \big|\mathbf{u}\big\rangle = 0. \tag{5.33}$$

In the absense of photons, the field part of the Hamiltonian vanishes, $H_A \big|\mathbf{u}\big\rangle = 0$, whereas

$$\begin{aligned} H_q^0 \big|\mathbf{u}\big\rangle &= M\gamma(\mathbf{u}) \big|\mathbf{u}\big\rangle, \\ H_q^1 \big|\mathbf{u}\big\rangle &= -i \sum_{\alpha=1}^{d-1} \int \frac{d^d k}{\sqrt{2|\mathbf{k}|}} \epsilon_{i\alpha}(\mathbf{k}) u_i \big|(\mathbf{k},\alpha); \mathbf{u}\big\rangle. \end{aligned} \tag{5.34}$$

There are two nonzero matrix elements of the Hamiltonian, with one state being the vacuum $\big|\mathbf{u}\big\rangle$:

$$\begin{aligned} \big\langle \mathbf{u} \big| H \big|\mathbf{u}'\big\rangle &= M\gamma(\mathbf{u})\delta(\mathbf{u}-\mathbf{u}'), \\ \big\langle (\mathbf{k},\alpha); \mathbf{u} \big| H \big|\mathbf{u}'\big\rangle &= -\frac{ie}{\sqrt{2|\mathbf{k}|}} u_i \epsilon_{i\alpha}(\mathbf{k})\delta(\mathbf{u}-\mathbf{u}'). \end{aligned} \tag{5.35}$$

The first equation asserts that the expectation value of the energy is $M\gamma(\mathbf{u})$, i.e. the energy of a point particle moving at speed $\mathbf{u}$. The second element is the amplitude for creating a one-photon state from the vacuum. It is nonzero provided that $u_i \epsilon_{i\alpha}(\mathbf{k}) \neq 0$, i.e. the photon's momentum must not be parallel to the observer's trajectory.

When the observer does not move, $u_i = 0$. The total energy of the vacuum $\big|\mathbf{0}\big\rangle$ thus equals the observer's mass M, as expected. The second matrix element in (5.35) vanishes. To have a nonzero amplitude for photon creation when $\mathbf{u} = \mathbf{0}$, we need to consider transition between one- and two-photon states. When $\mathbf{u} = \mathbf{0}$,

$$\begin{aligned} H_A \big|(\mathbf{k},\alpha); \mathbf{0}\big\rangle &= |\mathbf{k}| \big|(\mathbf{k},\alpha); \mathbf{0}\big\rangle, \\ H_q^0 \big|(\mathbf{k},\alpha); \mathbf{0}\big\rangle &= \sqrt{M^2 + |\mathbf{k}|^2} \big|(\mathbf{k},\alpha); \mathbf{0}\big\rangle, \\ H_q^1 \big|(\mathbf{k},\alpha); \mathbf{0}\big\rangle &= i \sum_{\beta=1}^{d-1} \int \frac{d^d k'}{\sqrt{2|\mathbf{k}'|}} \epsilon_{i\beta}(\mathbf{k}') \frac{k_i}{\sqrt{M^2 + |\mathbf{k}|^2}} \big|(\mathbf{k},\alpha), (\mathbf{k}',\beta); \mathbf{0}\big\rangle. \end{aligned} \tag{5.36}$$

Hence

$$\big\langle (\mathbf{k}',\beta);\mathbf{u}\big|\, H \,\big|(\mathbf{k},\alpha);\mathbf{0}\big\rangle = (M+\epsilon_{\mathbf{k}})\delta(\mathbf{k}-\mathbf{k}')\delta_{\alpha\beta}\delta(\mathbf{u}), \tag{5.37}$$

where

$$\epsilon_{\mathbf{k}} = |\mathbf{k}| + \sqrt{M^2+|\mathbf{k}|^2} - M \;\approx\; |\mathbf{k}| + \frac{|\mathbf{k}|^2}{2M} \tag{5.38}$$

is the QJT energy (4.44) of a single free photon. The transition amplitude from one to two photons is

$$\begin{aligned}
&\big\langle (\mathbf{k}_1,\beta_1),(\mathbf{k}_2,\beta_2);\mathbf{u}\big|\, H \,\big|(\mathbf{k},\alpha);\mathbf{0}\big\rangle \;= \qquad\qquad (5.39)\\
&= \frac{ie}{\sqrt{M^2+\mathbf{k}^2}}\bigg(\frac{1}{\sqrt{2|\mathbf{k}_1|}}\; k_i\epsilon_{i\beta_1}(\mathbf{k}_1)\delta(\mathbf{k}-\mathbf{k}_2)\delta_{\alpha\beta_2} + 1\leftrightarrow 2\bigg)\delta(\mathbf{u}).
\end{aligned}$$

Even in a frame where the observer's velocity is zero, there is a nonzero matrix element for creating a two-photon state from a one-photon state, because the electromagnetic field interacts with the observer. Note that the first term vanishes when $\mathbf{k}$ is parallel to $\mathbf{k}_1$. We will discuss the form of this amplitude further in section 7, and contrast it to the corresponding object in gravity.

6 Gravity

6.1 Action

Finally, we turn to gravity in four dimensions, described by the Einstein action

$$\begin{aligned}
S_G &= \frac{1}{16\pi G}\int d^4x\,\sqrt{\det g}\,R(g) \\
&= \frac{1}{2\lambda^2}\int d^4x\,\sqrt{\det g}\,R(g),
\end{aligned} \tag{6.1}$$

where G is Newton's constant and

$$\lambda = \sqrt{\frac{8\pi G\hbar}{c}} = \sqrt{8\pi}\ell_{Pl} \tag{6.2}$$

is a parameter of the order of the Planck length ℓ_{Pl}. In QJT, we must also describe the observer. The total action is thus $S = S_G + S_q$, where the

observer part S_q is the proper length in the presence of a non-flat metric $g_\mu(x)$:

$$S_q = -M \int dt \sqrt{g_{\mu\nu}(0)\dot{q}^\mu(t)\dot{q}^\nu(t)}. \tag{6.3}$$

Because we work with relative fields, it is $g_{\mu\nu}(0)$ rather than $g_{\mu\nu}(q(t))$ which appears in this formula.

We only consider linearized gravity, and thus assume that the metric can be written as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $h_{\mu\nu}$ is small compared to the Minkowski metric. Define the graviton field $\phi_{\mu\nu}$ by

$$\lambda\phi_{\mu\nu} = \bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h, \tag{6.4}$$

where indices are now raised and lowered by means of the Minkowski metric, and $h = \eta^{\mu\nu}h_{\mu\nu}$. The action (6.2) has a gauge symmetry of diffeomorphisms which allows us to eliminate eight of the ten components of $\phi_{\mu\nu} = \phi_{\nu\mu}$. We impose the following gauge conditions:

$$\begin{array}{rcll} \phi_{\mu 0} &=& 0, & \\ \partial_j\phi_{ij} &=& 0, & \text{(Lorentz gauge)} \\ \phi_{ii} &=& 0. & \text{(no spin 0)} \end{array} \tag{6.5}$$

The Einstein action can be separated into a free and an interaction part, $S_G = S_G^0 + \lambda S_G^1$. In this paper we will only be concerned with the free graviton action, which reads, with the gauge choices above,

$$\begin{aligned} S_G^0 &= \frac{1}{2}\int d^4x\, \partial_\rho\phi_{\mu\nu}\partial^\rho\phi^{\mu\nu} \\ &= \frac{1}{2}\int d^4x\, \big(\partial_0\phi_{ij}\partial_0\phi_{ij} - \partial_k\phi_{ij}\partial_k\phi_{ij}\big). \end{aligned} \tag{6.6}$$

In the same gauge, the observer action becomes

$$\begin{aligned} S_q &= -M\int dt\, \sqrt{1 - g_{ij}(\mathbf{0})\dot{q}_i(t)\dot{q}_j(t)} \\ &= -M\int dt\, \sqrt{1 - \dot{\mathbf{q}}^2 - \lambda\phi_{ij}(\mathbf{0})\dot{q}_i(t)\dot{q}_j(t)}. \end{aligned} \tag{6.7}$$

6.2 Hamiltonian

The canonical momenta are

$$\begin{aligned} \pi_{ij}(\mathbf{x}) &= \partial_0\phi_{ij}(\mathbf{x}), \\ p_i(t) &= \frac{M}{\dot\tau(t)} g_{ij}(\mathbf{0})\dot q_j + P_i, \end{aligned} \tag{6.8}$$

where $\partial_0 = \partial/\partial t + \dot q_i \partial/\partial x_i$ as in (4.4), and

$$\begin{aligned} P_i &= \int d^3x\, \partial_i\phi_{jk}(\mathbf{x})\partial_0\phi_{jk}(\mathbf{x}), \\ \dot\tau(t) &= \sqrt{1 - g_{ij}(\mathbf{0})\dot q_i(t)\dot q_j(t)} \end{aligned} \tag{6.9}$$

is the derivative of the proper time $\tau(t)$. The Hamiltonian is of the form $H = H_G^0 + \lambda H_G^1 + H_q$, where

$$\begin{aligned} H_G^0 &= \frac{1}{2}\int d^3x \left(\pi_{ij}(\mathbf{x})\pi_{ij}(\mathbf{x}) + \partial_k\phi_{ij}(\mathbf{x})\partial_k\phi_{ij}(\mathbf{x})\right), \\ H_q &= \frac{M}{\dot\tau} = \sqrt{M^2 + (\mathbf{p}-\mathbf{P})^2 + \lambda\phi_{ij}(\mathbf{0})(p_i - P_i)(p_j - P_j)}, \\ P_i &= \int d^3x\, \partial_i\phi_{jk}(\mathbf{x})\pi_{jk}(\mathbf{x}), \\ \dot\tau &= \frac{M}{\sqrt{M^2 + (\mathbf{p}-\mathbf{P})^2 + \lambda\phi_{ij}(\mathbf{0})(p_i - P_i)(p_j - P_j)}}, \end{aligned} \tag{6.10}$$

and the interaction Hamiltonian H_G^1 describes graviton-graviton scattering. These definitions of P_i and $\dot\tau$ are compatible with (6.9).

6.3 Oscillators

The gauge choice (6.5) means that the Fourier modes $\phi_{ij}(\mathbf{k})$ and $\pi_{ij}(\mathbf{k})$ are not independent; they are subject to the conditions

$$k_j\phi_{ij}(\mathbf{k}) = \phi_{ii}(\mathbf{k}) = k_j\pi_{ij}(\mathbf{k}) = \pi_{ii}(\mathbf{k}) = 0. \tag{6.11}$$

The CCR read

$$[\phi_{ij}(\mathbf{k}), \pi_{\ell m}(\mathbf{k}')] = \frac{i}{2}(\delta_{i\ell}\delta_{jm} + \delta_{j\ell}\delta_{im})\delta(\mathbf{k}+\mathbf{k}'), \tag{6.12}$$

up to terms needed ensure compatibility with the gauge conditions.

In analogy with the electromagnetic case (5.21), we introduce spin-2 polarization tensors $\epsilon_{ij\alpha}(\mathbf{k})$, $\alpha = 1, 2$, which satisfy

$$\begin{aligned} \epsilon_{ij\alpha}(\mathbf{k})\epsilon_{ij\beta}(-\mathbf{k}) &= \delta_{\alpha\beta}, & \epsilon_{ij\alpha}(\mathbf{k}) &= \epsilon_{ji\alpha}(\mathbf{k}), \\ k_j\epsilon_{ij\alpha}(\mathbf{k}) &= 0, & \epsilon_{ii\alpha}(\mathbf{k}) &= 0. \end{aligned} \tag{6.13}$$

The independent spin-2 oscillators, labelled by $\alpha = 1, 2$,

$$\begin{aligned} a_\alpha(\mathbf{k}) &= \frac{1}{\sqrt{2|\mathbf{k}|}}\epsilon_{ij\alpha}(-\mathbf{k})(\pi_{ij}(\mathbf{k}) - i|\mathbf{k}|\phi_{ij}(\mathbf{k})), \\ a^\dagger_\alpha(\mathbf{k}) &= \frac{1}{\sqrt{2|\mathbf{k}|}}\epsilon_{ij\alpha}(-\mathbf{k})(\pi_{ij}(\mathbf{k}) + i|\mathbf{k}|\phi_{ij}(\mathbf{k})), \end{aligned} \tag{6.14}$$

are subject to the nonzero CCR

$$[a_\alpha(\mathbf{k}), a^\dagger_\beta(\mathbf{k}')] = \delta_{\alpha\beta}\delta(\mathbf{k} + \mathbf{k}'). \tag{6.15}$$

The free field part of the Hamiltonian becomes a sum over non-interacting gravitons,

$$H^0_G = \int d^3x\, |\mathbf{k}| a^\dagger_\alpha(\mathbf{k}) a_\alpha(-\mathbf{k}), \tag{6.16}$$

whereas the observer part can be expanded in a power series in λ, $H_q = H^0_q + \lambda H^1_q + O(\lambda^2)$, where

$$\begin{aligned} H^0_q &= \sqrt{M^2 + (\mathbf{p} - \mathbf{P})^2} \equiv \sqrt{M^2 + (p_i - P_i)(p_i - P_i)}, \\ H^1_q &= \frac{1}{2H^0_q}\phi_{ij}(\mathbf{0})(p_i - P_i)(p_j - P_j). \end{aligned} \tag{6.17}$$

Again we have an ordering ambiguity because the graviton oscillators do not commute with

$$P_i = \int d^3k\, k_i a^\dagger_\alpha(\mathbf{k}) a_\alpha(-\mathbf{k}). \tag{6.18}$$

We choose to normal order the interaction Hamiltonian, i.e.

$$\begin{aligned} H^1_q = i\sum_{\alpha=1}^{2}\int \frac{d^3k}{\sqrt{2|\mathbf{k}|}}\epsilon_{ij\alpha}(\mathbf{k})\Big(&\frac{(p_i - P_i)(p_j - P_j)}{2H^0_q}a_\alpha(\mathbf{k}) - \\ &- a^\dagger_\alpha(\mathbf{k})\frac{(p_i - P_i)(p_j - P_j)}{2H^0_q}\Big). \end{aligned} \tag{6.19}$$

This Hamiltonian is very similar to the electromagnetic Hamiltonian in (5.27) and (5.29), except that the spin-2 gravitons interact with the tensor $(p_i - P_i)(p_j - P_j)$ rather than the vector $p_i - P_i$.

6.4 Quantization

We quantize the theory in complete analogy with the electromagnetic field in the previous section. The Hamiltonian has the following nonzero matrix elements with the vacuum $\big|\mathbf{u}\big\rangle$, where the observer moves with velocity $\mathbf{u}$:

$$
\begin{aligned}
\big\langle \mathbf{u}\big| H \big|\mathbf{u}'\big\rangle &= M\gamma(\mathbf{u})\delta(\mathbf{u}-\mathbf{u}'), \\
\big\langle (\mathbf{k},\alpha);\mathbf{u}\big| H \big|\mathbf{u}'\big\rangle &= -\frac{i\lambda}{2M\sqrt{2|\mathbf{k}|}} u_i u_j \epsilon_{ij\alpha}(\mathbf{k})\delta(\mathbf{u}-\mathbf{u}').
\end{aligned}
\tag{6.20}
$$

The first element is the energy of the moving observer itself, and the second element vanishes if $\mathbf{u}$ is parallel to $\mathbf{k}$, since $k_j\epsilon_{ij\alpha}(\mathbf{k}) = 0$. In particular, the amplitude vanishes for a still-standing observer.

The expectation value of the energy in a one-graviton state, $\big\langle (\mathbf{k}',\beta);\mathbf{u}\big| H \big|(\mathbf{k},\alpha);\mathbf{0}\big\rangle$, has the same value (5.37) as in a one-photon state. To compute the transition amplitude between one and two photons, we need

$$
\begin{aligned}
&H^1_q \big|(\mathbf{k},\alpha);\mathbf{0}\big\rangle = \\
&= \frac{i}{2\sqrt{M^2+\mathbf{k}^2}} \sum_{\alpha'=1}^{2} \int \frac{d^dk'}{\sqrt{2|\mathbf{k}'|}} k_i k_j \epsilon_{ij\alpha'}(\mathbf{k}') \big|(\mathbf{k},\alpha),(\mathbf{k}',\alpha');\mathbf{0}\big\rangle.
\end{aligned}
\tag{6.21}
$$

Hence the transition amplitude is

$$
\begin{aligned}
&\big\langle (\mathbf{k}_1,\beta_1),(\mathbf{k}_2,\beta_2);\mathbf{u}\big| H \big|(\mathbf{k},\alpha);\mathbf{0}\big\rangle = \\
&= \frac{i\lambda}{2\sqrt{M^2+\mathbf{k}^2}} \bigg(\frac{1}{\sqrt{2|\mathbf{k}_1|}}\ k_i k_j \epsilon_{ij\beta_1}(\mathbf{k}_1)\delta(\mathbf{k}-\mathbf{k}_2)\delta_{\alpha\beta_2} \\
&\qquad + 1 \leftrightarrow 2 \bigg)\delta(\mathbf{u}).
\end{aligned}
\tag{6.22}
$$

The first term vanishes if $\mathbf{k}$ is parallel to $\mathbf{k}_1$, since $k_j\epsilon_{ij\alpha}(\mathbf{k}) = 0$.

7 Rescaling and discussion

To extract how matrix elements depend on the observer's mass, it is useful to isolate the M dependence. Introduce the dimensionless momentum κ by setting $\mathbf{k} = M\kappa$, and rescale all other quantities with appropriate powers of M. If the engineering dimension $[\psi(\mathbf{k})] = D$, define the dimensionless quantity $\psi(\kappa)$ by $\psi(\mathbf{k}) = M^D\psi(\kappa)$. The engineering dimensions of the fields

in d dimensions are

$$\begin{array}{ll} [\mathbf{x}] = -1 & [\mathbf{k}] = +1 \\ [d^d x] = -d & [d^d k] = +d \\ [\phi(\mathbf{x})] = (d-1)/2 & [\pi(\mathbf{x})] = (d+1)/2 \\ [\phi(\mathbf{k})] = -(d+1)/2 & [\pi(\mathbf{k})] = (1-d)/2 \\ [a_\alpha(\mathbf{k})] = -d/2 & [a^\dagger_\alpha(\mathbf{k})] = -d/2 \\ [\delta(\mathbf{x})] = d & [\delta(\mathbf{k})] = -d \\ [H] = +1 & [\mathbf{u}] = 0 \end{array} \tag{7.1}$$

The free-field Hamiltonian given by the following expression:

$$\begin{aligned} H_\phi &= \frac{M}{2}\int d^d\kappa \left(\pi(\kappa)\pi(-\kappa) + \omega_\kappa^2 \phi(\kappa)\phi(-\kappa)\right) \\ &= M\int d^d\kappa\, \omega_\kappa a^\dagger(\kappa)a(-\kappa). \end{aligned} \tag{7.2}$$

The energy of the free-field n-quanta state, when the observer's velocity is zero, reads

$$\begin{aligned} H &= M\bigg(\sum_\ell \omega(\kappa_\ell) + \sqrt{1 + (\sum_\ell \kappa_\ell)^2}\bigg) \\ &\approx M\bigg(1 + \sum_\ell \epsilon_{\kappa_\ell} + \sum_{\ell<\ell'} \kappa_\ell \cdot \kappa_{\ell'}\bigg), \end{aligned} \tag{7.3}$$

where

$$\epsilon_\kappa = \omega_\kappa + \frac{1}{2}\kappa^2 \tag{7.4}$$

is the energy of a single quantum. The use of relative fields thus results in two effects.

1. The single-quantum energy (7.4) acquires a shift $\kappa^2/2$.
2. There is an interference term in (7.3), which originates from the shift $\mathbf{p} \to \mathbf{p} - \mathbf{P}$ in the observer's momentum.

Both these effects vanish in the limit $\kappa \to \mathbf{0}$, i.e. $M \to \infty$, and the QJT energy reduces to the QFT energy in this limit. The observer's mass thus effectively becomes a cutoff, below which QJT reduces to QFT.

In electromagnetism and gravity we also introduced an explicit observer-field interaction in the action. Such an interaction could of course also be

introduced within the framework of absolute fields, but the motivation in QJT is much stronger because the observer's position is already present in the definition of ∂_0, and as a quantum observable it must obey some dynamics. The new term causes the fields to interact with the observer, giving nonzero matrix elements between states with different numbers of quanta. In particular, when all powers of M have been extracted, the matrix element in (5.39) becomes

$$\big\langle(\kappa_1,\beta_1),(\kappa_2,\beta_2);\mathbf{u}\big|\, H\,\big|(\kappa,\alpha);\mathbf{0}\big\rangle = eM^{\frac{d-1}{2}}F\delta(\mathbf{u}), \tag{7.5}$$

where

$$\begin{aligned} F &\equiv F((\kappa_1,\beta_1),(\kappa_2,\beta_2),(\kappa,\alpha)) \\ &= \frac{i}{\sqrt{1+\kappa^2}}\bigg(\frac{1}{\sqrt{2|\kappa_1|}}\,\kappa_i\epsilon_{i\beta_1}(\kappa_1)\delta(\kappa-\kappa_2)\delta_{\alpha\beta_2}+1\leftrightarrow 2\bigg) \end{aligned} \tag{7.6}$$

is a dimensionless number. The analogous amplitude (6.22) in four-dimensional gravity is

$$\big\langle(\kappa_1,\beta_1),(\kappa_2,\beta_2);\mathbf{u}\big|\, H\,\big|(\kappa,\alpha);\mathbf{0}\big\rangle = \lambda M^2 G\delta(\mathbf{u}), \tag{7.7}$$

where

$$\begin{aligned} G &\equiv G((\kappa_1,\beta_1),(\kappa_2,\beta_2),(\kappa,\alpha)) \\ &= \frac{1}{2\sqrt{1+\kappa^2}}\bigg(\frac{1}{\sqrt{2|\kappa_1|}}\,\kappa_i\kappa_j\epsilon_{ij\beta_1}(\kappa_1)\delta(\kappa-\kappa_2)\delta_{\alpha\beta_2} \\ &\qquad +1\leftrightarrow 2\bigg) \end{aligned} \tag{7.8}$$

is also a dimensionless number. Let us introduce an abbreviated notation, where $\big|n\big\rangle$ stands for an n-quanta state, and the momenta and polarizations are implicit. The transition amplitude from one to two quanta can then be written in $d=3$ as

$$\begin{aligned} \big\langle 2\big|\, H\,\big|1\big\rangle &= eMF\delta(\mathbf{u}), && \text{(EM)}, \\ \big\langle 2\big|\, H\,\big|1\big\rangle &= (\lambda M)MG\delta(\mathbf{u}), && \text{(Gravity)}. \end{aligned} \tag{7.9}$$

Since the engineering dimension of the Hamiltonian equals one, the rescaled matrix elements are proportional to the dimensionless numbers e and λM, respectively.

We may now make some general observations about the consequences of relative fields.

1. QFT, or at least the correct energy levels, is recovered from QJT in the limit $M \to \infty$. Since this assumption is incompatible with virtual quanta with energy $E > M$, the $M \to \infty$ limit of QJT is essentially QFT with cutoff scale M.

2. In electromagnetism, the field-observer interaction gives nonzero matrix elements between states with different numbers of photons. Since the amplitudes are proportional to the observer's charge e, this effect vanishes in the limit $e \to 0$.

3. We expect other nongravitational interactions to behave similarly to electromagnetism. In particular, the effect should vanish in the limit that the observer is uncharged.

4. The corresponding matrix element in gravity is proportional to λM. Since the Planck length λ is a universal constant, graviton number is conserved by the observer-field interaction in the limit $M \to 0$. However, this limit is incompatible with the assumption $M \to \infty$ made at point 1 above. Hence QJT does not possess a QFT limit specifically for gravity.

Only linearized gravity was considered in this paper. The interaction part of the Hamiltonian H^1_G causes graviton-graviton interactions and problems with infinities. There is no obvious reason why passing to relative fields should improve the situation. QJT modifies the dispersion law for energetic quanta, but the Hamiltonian (7.3) still grows linearly for large momenta.

However, it seems plausible that one can construct a model where divergent contributions from bosonic and fermionic fields cancel. This hope emanates from the construction in [2, 3, 4], where a recipe for cancelling divergent contributions to diff anomalies was developed, leaving only a finite cocycle in the limit of infinite jets.

8 Conclusion

Quantum Jet Theory is an UV completion of QFT; more precisely, it is the deformation of QFT whose deformation parameter is the observer's mass M. QFT is recovered from QJT in the limit $G = 0$, $M \to \infty$, and general relativity is recovered in the limit $\hbar = 0$, $M \to 0$. Since these limits are mutually incompatible, no QFT description of gravity is possible. To construct a consistent quantum theory of gravity, we expect that QJT is needed, with a finite, nonzero observer mass.

It was noted in subsection 3.6 that observer dependence is not the same as frame dependence. We typically work in the frame of GPS satellites; spacetime points are labelled by their GPS coordinates. In contrast, observer dependence enters "at the other side of the measuring rod". In QJT, it is the distance between the point (the reading of a GPS device) and the physical observer that is the partial observable, but in QFT it is the point itself. This leads to a shift in the observer's momentum: $\mathbf{p} \to \mathbf{p} - \mathbf{P}$, where $\mathbf{P}$ is the momentum of the fields. We saw in equation (4.14) that $\mathbf{P} = 0$ classically, at least for the free scalar field and probably in general. This means that using relative fields only matters on the quantum level; the classical limits of QJT and QFT are the same.

As is discussed in detail in the companion paper [5], QJT leads to new gauge and diff anomalies not present in QFT. This unambigously proves that QJT is substantially different from QFT, which is positive given that QFT is incompatible with gravity. The presence of a diff anomaly invalidates standard claims that gravity must be holographic; it is well known from CFT that diffeomorphism symmetry on the circle is compatible with nontrivial correlators, but only in the presence of a Virasoro central charge.

QJT is new physics in the sense that its predictions differ from those of QFT, but it does not add new terms to the Lagrangian. However, several major discoveries during the past century (special relativity, quantum mechanics, renormalization) were not primarily about new terms in the Lagrangian, but rather about new ways thinking about observers and observability. QJT takes one step further in this direction by upgrading the observer to a physical actor with quantum dynamics.

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